

The Emergence of Modern Number Theory in Hungary

Michael J. Coons¹
Alfréd Rényi Institute
Hungarian Academy of Sciences
H-1053 Budapest
Hungary
michael_coons@baylor.edu

Abstract

The goal of this paper is to present an introduction to the emergence of modern number theory in Hungary by recounting the lives of some of its most influential mathematicians, and examining developments in the mathematical life of Hungary in the nineteenth century. Special attention is given to the predecessors of Pál Erdős and Pál Turán, including János Bolyai, Gyula Kőnig, Gusztáv Rados, and Mihály Bauer, and to the mathematical publications and societies they were instrumental in founding.

Preface

Hungarian mathematics became famous in the twentieth century due to the outstanding results of a handful of very talented and hard-working individuals, such as Pál Erdős, Alfréd Rényi, Pál Turán, and János von Neumann. The emergence of this group of mathematicians brought an onslaught of number-theoretic results, putting Hungary on the modern mathematical map, as well as securing a spot in history for Hungarian mathematicians. In recognition of the many wonderful results of Hungarian mathematicians, their most influential work has been compiled and released by the Bolyai Mathematical Society in a two volume set, *A Panorama of Hungarian Mathematics in the Twentieth Century I, II*² [5]. However, there is still a shortage of information on the predecessors of Hungary's modern mathematical giants.

Having been steeped in mathematics and its culture for the last six years as a student, I have developed a strong interest in the history of mathematics, specifically, the general trends in number-theoretic research. I have become acquainted with many of the works of Erdős, Rényi, Turán, and their contemporaries, which directed my attention to Hungary.

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²The first of these two volumes is available, and the second is currently being written.

With number-theoretic contributions as large as the ones made by Hungarians in the twentieth century, I became curious about the predecessors of such an influential group. Who were they? What were they interested in? What questions did they ask? What questions did they answer? What is so special about mathematics in Hungary? Where did these mathematicians really come from?

These questions brought me to Budapest to search out the history of the enduring legacy of Hungarian number theory.

Within a week or two of arrival, I had been welcomed into the math branch of the Hungarian Academy of Sciences, the Alfréd Rényi Institute, which houses the offices and meeting halls of dozens of the country's top mathematicians. I met János Pintz and Andras Biró, who were extremely helpful to me in my studies of prime number theory, and began attending the weekly number theory seminar. In addition to these live sources, the Institute also contains a fantastic library that would become the home of my quest for historic information.

At the beginning of my stay in Hungary, I also enrolled in a language course to help me get by in my everyday life in Budapest that would prove useful to my research later. Since there was no language requirement for the Fulbright fellowship, and since all of the mathematics that I was acquainted with was published in English, French, or German (all languages I am comfortable with), or even Latin for the earlier works, I thought that I was linguistically prepared for my research. This did not turn out to be the case.

Walking through the aisles of the Rényi Library, I saw shelves and shelves of books that were written in neither English, nor French or German. There were hundreds of books in Hungarian, Russian, Polish, and various other languages that I couldn't even recognize.

The library itself is something of a maze. In the entry, there is a wall of reference books on the left and a couple of computers and a printer on the right. Where the reference books end, a passageway leads to a reading room, with a very large table in it, and a set of shelves with some of the newer journals on display. The passageway houses at least two distinct card catalogues. I'm pretty sure that there are three, but the third one is very small, and may only be a supplement to the larger two. On the right, directly after the group of computers are a couple of stairs which lead into the book room. The book room is large, with extremely high ceilings. The vertical space is split into two levels, housing the journals on the bottom floor and the books on the top floor, with a layer of Art-Deco style glass bricks hovering in between. Leading to the upper level, all the way to the back of the journal floor, between yet another card

catalogue (for journals only) and a wall of what look to be stacks of manuscripts, there is a set of fire-escape style stairs. As I surveyed the collection, it became apparent that the Dewey Decimal system is not in effect here. As far as I could tell, when a new book arrives, the librarian gives it the next available number and then places it at the end of the shelf. Multiple copies of a certain book are almost always in different locations.

Overwhelmed by my initial visit, I withdrew to reevaluate my approach.

After a few months of language study, I tried the library again, and began to gather information on Hungarian-language number theory. Even with my rudimentary language skills, the library at the Rényi Institute yielded treasure.

Finding a book at the Rényi Library can be a difficult task. After much fruitless wandering, I became completely dependent on the three (or is it four?) card catalogues and the computer catalogue. At one point in my information collection stage, I spent a whole day looking through forty years worth of *Mathematikai Lapok*. This process entailed me going through a large shelf full of journals one by one, browsing the titles of the articles for hints of something that might be useful in my study. Sifting through *Mathematikai Lapok*, I found a handful of useful articles to supplement my other finds. Old documents I couldn't locate, the helpful and all-knowing librarian would ferret out of storage in another part of the Institute. Rumor has it that the Institute has been buying up neighboring flats and filling them with books, not even my advisor, János Pintz, knows how many or where. I imagine troves of old notebooks behind locked doors, hidden passages lined with manuscripts, mountains of books occupying abandoned kitchens and bedrooms, just waiting to be explored.

In the paper that follows, I share the answers to as many of my questions as I could find in these few months in Budapest.

Introduction

With the bicentennial anniversary of the birth of János Bolyai in 2002, his manuscripts, kept in the Teleki-Bolyai Library at Marosvásárhely (Transylvania), were released for viewing to historians of mathematics. In these manuscripts, the historians discovered previously unknown number theory notes (see [6, 9]). Until this point all of Bolyai's number-theoretic ideas went unpublished, and thus unknown for the better part of two hundred years. Due to this lack of published number-theoretic results, Bolyai played little to no part in the devel-

opment of the subject in Hungary³, though he was the first Hungarian mathematician to produce significant results in number theory. It can only be left to speculation what would have happened if Bolyai would have communicated his number theory to the next generation of Hungarian mathematicians.

During the last half of the nineteenth century, in Budapest Gyula Kónig and Gusztáv Rados were helping to develop a solid foundation in both mathematics teaching and number-theoretic research. Gyula Kónig, especially, has become known as one of the outstanding pedagogues of this time period, being one of the first to bring Hungarian mathematics teaching to the foreground of pedagogy.

Gusztáv Rados helped start mathematical societies and journals, as well as teaching and doing research of his own. Rados was one of the founders of the Mathematical and Physical Society, which is now known as the Bolyai Mathematical Society, in honor of János Bolyai.

A student of Gusztáv Rados, Mihály Bauer, was one of the first true Hungarian number theorists. His commitment to research and passion for teaching played an important role in the mathematical life of Hungary, and it is Bauer who provides a crucial link to the well known Hungarian mathematical legacies of Pál Erdős and Pál Turán.

The Language of Mathematics in Hungary

Though Hungarian is spoken in Hungary's former territories in neighboring countries, and by pockets of immigrants in Ohio, the Hungarian language is an island. It is not linguistically related to any of its neighbors. Because of this, Hungarians have learned the languages of their neighbors, and in terms of mathematics or science, if they wish to share their results with a larger community, this means publishing in an internationally understood language.

Before the year 1800, Latin was the primary language of mathematics publications, though German was rising in popularity in Hungary as early as the mid-1700s.

Sometime around the middle of the nineteenth century, Hungarian mathematicians cut back on publishing in German and Latin, and made a great effort to publish and write in their native Hungarian. János Bolyai, in one of his

³Though Bolyai did not play a large role in building a number-theoretic foundation in Hungary, the town of Marosvásárhely would not go unrepresented in this area in the nineteenth century. Gyula Vályi, a Marosvásárhely native, contributed significantly to the teaching of mathematics and specifically to the advancement of number theory through his lectures at the University of Kolozsvár. I don't give an account of Gyula Vályi in this article, but for those interested, the book [23] gives a good overview of his life and work.

notebooks illustrates this trend, by stating that “since 1842 (writing in German till that time)” he began to turn his “attention more seriously to Hungarian as a language, whose foundations were built in [his] youth ([6], p. 54).”

This shift toward using the common language was mainly due to the 1825 formation of the Hungarian Scholarly Society, which would later become the Hungarian Academy of Sciences, by István Széchenyi. Between 1825 and 1830, a great reform movement⁴ began in Hungary, headed mainly by István Széchenyi, who is credited for the 1844 establishment of Hungarian as the official national language (which was previously Latin). “The question of cultivating and popularizing the sciences in the Hungarian language was a central concern of the members of the Academy from the very beginning, so they launched a periodical⁵ in anticipation of a wide readership in 1834 ([21], p. 198).”

As scholarly life developed in Hungary, no fewer than seven mathematical journals were founded (see Table refjournaltable), although few of them survived the turn of the century.

Table 1: 19th century Hungarian mathematical journals

1834	Tudománytár
1840	Magyar Académiai Értesítő
(?)1844	Mathematicai Pályamunkák
1882	Mathematikai és Természettudomány Értesítő
1883	Mathematische und naturwissenschaftliche Berichte aus Ungarn
1891	Mathematikai és Fizikai Lapok
1894	Középiskola Mathematikai Lapok

In an attempt to standardize Hungarian mathematical terms, the Academy published a mathematics dictionary (*Mathematikai Műszótár*) in 1834. Though the gesture and motivation of producing a mathematics dictionary was well intended, the dictionary itself turned out to be somewhat of a disappointment. The most familiar Hungarian mathematics terms, many of which are still in use today, come from the widely used 1861-1862 two volume textbook of János Ármin Vész [22].

Included in this age of academic reform, the Hungarian Scholarly Society published a Hungarian translation of Euclid’s Elements in 1832, released a series

⁴For more information see pages 191-205 in [15].

⁵Tudománytár

of books, entitled *Értekezések a Matematikai Tudományok Köréből*⁶ between 1867 and 1894, and in 1891 the journal *Mathematikai és Fizikai Lapok*⁷ was founded.

Though Hungarian mathematicians would continue to publish in the Hungarian language, they realized publication strictly in Hungarian would be to their detriment at the international level, and in 1883 the German language journal *Mathematische und naturwissenschaftliche Berichte aus Ungarn* was founded. Because of personal, geographical, and political ties to the German speaking world, *Ungar. Berichte* gained international popularity, and was widely read.

Early Investigations: János Bolyai

Until recently, serious research in number theory was thought to have begun in Hungary during the last part of the nineteenth century. In his book, *History of Mathematics in Hungary until the 20th Century*, Barna Szénássy writes, “Up to the last quarter of the 19th century, no noteworthy results in number theory were produced in Hungary. We know from the writings of the two Bolyai’s that they has a high esteem for the “queen” of mathematics⁸, yet their posthumous papers contained scarcely any ideas in this field ([21], p. 239).”

The recent release of János Bolyai’s notebooks from the Teleki-Bolyai Library brought new information to light. Under careful scrutiny by mathematics historians such as Elemér Kiss, Bolyai’s notebooks revealed a tremendous amount of number-theoretic results. This is surprising, considering that up to this point “all Bolyai-monographs unanimously assert: although János Bolyai tried his hand at a few problems in number theory, his investigations were not particularly successful ([7], p. 6).”

János Bolyai and his father, Farkas, are widely known in Hungary, for their mathematics as well as through the plays and books written about their family and relationships. The Bolyais have also been the object of many critical and scandalous stories, which have made them famous.

⁶This book seires aong with *Mathematikai és Természettudomány Értesítő* were only published in Hungarian. They mode no exception for other languages, including abstracts.

⁷The contents of this journal have become valuable to my studyof Hungarian mathematics. Though a lot of very important results would continue to be published in various international languages, this journal would be a turning point for publications of mathematics in the Hungarian language. It continued to be published well into the twentieth century.

⁸This is number theory’s nickname, due to Gauss’ quote, “Mathematics is the queen of the sciences and number theory is the queen of mathematics. She often condescends to render service to astronomy and other natural sciences, but in all relations she is entitled to the first rank.”

János Bolyai was born on December 15th, 1802 in Kolozsvár (now Cluj, Romania). He was taught mathematics by his father, who was a professor at the Calvinist College in Marosvásárhely. When János was fourteen, his father wrote to his friend Carl Gauss to request that János study under him at Göttingen. Farkas' letter yielded no response, and in 1818 János began studying at the Royal Engineering College in Vienna. He graduated in 1822, three full years early, and spent the next eleven years in military service.

Throughout this time János communicated regularly with his father, with mathematics the subject of almost every letter.

While Farkas Bolyai published quite a bit of mathematics, his son János did not. He did produce two famous works in the *Responsio* and what is known as the *Appendix*, as it was published as such in the *Tentamen*, a work of his father's. The *Appendix*, whose actual title is *The Absolutely True Science of Space*, is the more famous of the two, and it was János Bolyai's only work published during his lifetime. In it, János Bolyai introduces what has come to be known as absolute geometry, that is, geometry without Euclid's axiom V, which states that parallel lines have no intersection.

Although János Bolyai is known for his work in geometry, he made numerous investigations in number theory, though none were published. Among these investigations, Bolyai pursued many of the same avenues as Pierre de Fermat; specifically, Bolyai had a strong interest in things related to Fermat's Little Theorem.

Theorem 1 (Fermat's Little Theorem). *Let p be a prime and suppose that $(a, p) = 1$. Then $a^{p-1} \equiv 1 \pmod{p}$.*

Proof of the converse of Fermat's Little Theorem would be very valuable, as it would give an exact formula for prime numbers. While trying to prove the converse of Fermat's Little Theorem, Bolyai discovered the smallest pseudoprime, 341, thus showing by counterexample that the converse is false. A pseudoprime is, by definition, a composite number satisfying Fermat's Little Theorem.

In this same vein, Bolyai's notes contain the proof of the Jeans' Theorem. While Bolyai did not publish his result, in 1898, decades after Bolyai wrote it in his notebooks, James Hopwood Jeans proved and published the same result, so this theorem is known usually as Jeans' Theorem.

Theorem 2 (Jeans Theorem). *Let p and q be primes, and let a be a natural number such that $(a, p) = 1$ and $(a, q) = 1$. If*

$$a^{p-1} \equiv 1 \pmod{q} \quad \text{and} \quad a^{q-1} \equiv 1 \pmod{p},$$

then $a^{pq-1} \equiv 1 \pmod{pq}$.

Among other pursuits, Bolyai developed, independently of Carl Gauss, the theory of complex integers, including divisibility and complex primes. A complex integer is an integer of the form $a + ib$, where both a and b are integers. These are referred to as Gaussian integers, named after Gauss, who published a theory concerning them. Using this theory, Bolyai produced a beautiful short proof to Fermat's Two-Squares Theorem (see [8]).

Theorem 3 (Fermat's Two-Squares Theorem). *Let p be a prime such that $p \equiv 1 \pmod{4}$. Then p can be written as the sum of two squares.*

Bolyai was also interested in Fermat and Mersenne Numbers, as well as the density of the primes. With all of this number theory appearing in the pages of János Bolyai's work, the older viewpoint that, "up to the last quarter of the 19th century, no noteworthy results in number theory were produced in Hungary ([21], p. 239)," is put to rest by Elemér Kiss. "We can assert that the first Hungarian mathematician to produce significant achievements in the field of number theory was János Bolyai ([7], p. 8)."

Unfortunately none of Bolyai's number-theoretic results were published. In fact the name of János Bolyai "never appeared in the publications of the Academy in his lifetime ([21], p. 199)." Due to his lack of published results, and because he was not involved with students, Bolyai would have almost no impact on the mathematics that took place in the years following his death in 1860.

Building a Foundation: König and Rados

Toward the end of the nineteenth century, a surge of educational reform took place in Hungary. In Budapest, the growth of science created a need for a central institution, so in 1856 the Joseph Industrial School became the Polytechnical School, which became the Technical University in 1871.

Soon after this, at the age of 25, Gyula König was appointed professor at the Technical University. König was born in Győr December 16, 1849. After pursuing a medical degree, he found himself taken by mathematics and earned his doctoral degree in Heidelberg in 1870, writing his dissertation on the theory of modular functions. After Heidelberg, König went to Berlin, where he studied under Kronecker and Weierstrass. With all of his education finished, and some experience working under a few of the giants of German mathematics, König returned to Hungary, taking a position at the Teacher's College of Budapest, and then at the Technical University.

One of Kőnig's first number-theoretic results was a theorem concerning the sums of the divisors of an integer (see [10]).

Theorem 4. *Let p be a prime dividing n . Then*

$$\sum_{d|n} d - \sum_{\substack{d|n \\ (d,p)=1}} d = p \left(\sum_{d|n} d - \sum_{\substack{d|n \\ (d,p)=1}} \frac{n}{d} \right).$$

It is worthwhile to note that Kőnig also published a proof of the Reciprocity Theorem by complete induction (see [11]).

Kőnig turned out not only to be a proficient researcher of mathematics, but an innovative educator involved in educational reform at not only the university level, but at the secondary school level as well, writing a textbook for the national secondary school algebra program.

Gusztáv Rados, one of Kőnig students, who later became his colleague at the Technical University, published a joint work with Kőnig on the theory of gamma functions [12], utilizing the simple inequality

$$0 \leq e^{-nx} - (1-x)^n < \frac{1}{n}$$

to avoid the cumbersome integrals that made the theory so computationally difficult.

Rados and Kőnig also have a theorem named after them which deals with the number of incongruent solutions to a polynomial modulus a prime (see [17]).

Theorem 5 (Kőnig-Rados Theorem). *Let $f(x) = \sum_{i=0}^{p-2} A_i x^i$, and let C be the cyclic coefficient matrix. Then $f(x) \equiv 0 \pmod{p}$ has $p-1-r_p$ incongruent solutions, if and only if,*

$$\text{rank}(C) \equiv r_p \pmod{p}.$$

Kőnig became a member of the Academy in 1889, before Rados, who became a corresponding member in 1894 and an ordinary member in 1907. Both Kőnig and Rados were key people in the establishment of the Mathematical Society, which was at first a very informal group of mathematicians that would get together to discuss current research, but later turned into a formal society. The Mathematical Society later became the Bolyai Mathematical Society. Kőnig and Rados also helped found the journal, *Mathematikai és Fizikai Lapok* in 1891.

An Heir: Mihály Bauer

Through their lectures at the Technical University, as well as through the Mathematical Society, Gyula Kőnig and Gusztáv Rados influenced many mathematicians and educators in Hungary. One such figure, who is central to this investigation of number theory in Hungary and a student of Rados, is Mihály Bauer. Like his mentors, mathematically, Bauer was open to new developments and techniques. He also researched prolifically, publishing 106 original works. Bauer's publications cover various areas of pure mathematics, including elementary number theory, congruence identities, factorization of polynomials, algebraic number theory, classical ideal theory, problems of complex numbers, p -adic numbers, characteristic polynomials, the number of subgroups of finite groups, theorems of irreducibility, and many other subareas of modern algebra. Despite his contributions to the field, Bauer's life and work have gone practically unnoticed.

Mihály Bauer was born on September 20, 1874 in Budapest, the city he would live in almost all of his life. Bauer started his mathematical research early, publishing his first paper [2] in 1892, under the tutelage of Gusztáv Rados. Bauer began to publish regularly in 1894. The quality of his research gained him fast attention from the mathematical community, though most of it would be from international mathematicians and not from within Hungary. Also in 1894, Bauer was given an 800Ft scholarship for the 1894-95 academic year at the Technical University, where six years later he was hired as an adjunct professor. His early influences include Dedekind, Hilbert, Kronecker, and Hansel, whose work Bauer developed further.

Most of Bauer's research focused on algebraic and elementary number theory, as well as function theory. Early in his career, Bauer was interested in Dirichlet's theorem for arithmetic progressions, and published [4] a very short elementary proof to the special case for progressions of the form $an + 1$.

Theorem 6 (Dirichlet). *Let $(a, b) = 1$. Then there are infinitely many primes in the arithmetic progression $an + b$ ($n=1, 2, 3, \dots$), with $(a, b) = 1$.*

Bauer's interest continued in this direction and in 1896, he published *Number theoretic theorems* [4], dealing with more issues of progressions, including proof of Theorem 7.

Theorem 7. *[[4] p. 149] Let k, l , and m be integers such that $(k, m) = d$ and*

$(k, l, m) = 1$, then the progression

$$\begin{aligned} &kn + l \pmod{m} \\ &(n = 1, 2, \dots, m) \pmod{m} \end{aligned}$$

contains $\frac{\varphi(m)}{\varphi(d)}$ numbers relatively prime to m .

As shown in the case of Bauer's proof of Dirichlet's Theorem, Bauer had a lot of success in simplifying the proofs of already existing theorems. He was also successful at widening the range of previous solutions.

Mihály Bauer was not only involved in mathematical research, but like his mentors, Kőnig and Rados, he cared about education as well, and in 1909 gained his instructor certification. He earned his regular teacher status in 1918, though he remained an adjunct at the Technical University. Bauer taught university lectures at a very high standard, focusing his lectures in algebraic number theory, algebra, real and complex analysis, and analytic number theory. Due to his many achievements, in 1922, Bauer was awarded the first Kőnig Prize, named after Gyula Kőnig.

In 1924, having been passed over for promotion at the Technical University, Bauer started teaching high school to ensure himself a pension for retirement. Though he enjoyed it, teaching was not easy for him. Despite his excellent teaching ability, Bauer's students, influenced by the politics of the time, frequently hassled him because he was Jewish.

The years preceding World War II were hard on Bauer. "The anti-Semitic agitation between 1919 and 1945 (its roots were clearly recognizable much earlier) sought to belittle the degree of the Jews' assimilation and their profound emotional loyalty to the Hungarian Fatherland by vague and manipulative concepts such as the "deep Magyar race", the "Magyar soul" the "Magyar nation" and the "Magyar genius", and to discredit the loyalty of outstanding figures of cultural and scientific life as "mimicry" ([15], p. 334)."

At the same time that he was begin admired by the international intellectual community, he was being stripped of his human rights by the Hungarian authorities. In this time of rising political and social unrest, Bauer decided to retire in 1936. His publication record also decreased in 1937 and ended completely in 1940, though he remained very sharp-witted.

Throughout this period, Bauer kept preparing lectures in the hope of teaching again. Sadly, this would not happen, though he did pass his lectures on to some of his students: Erdős, Turán, and Sándor. These three students studied privately under Bauer, having been totally separated from the university because of their Jewish heritage.

In a 1944 letter to László Rédei, one of his students, Bauer explained that the Hungarian authorities had dragged him into the ghetto. Bauer survived there until the liberation in January of 1945, only to fall a month later, receiving a concussion which would take him to the hospital, where he died in February 1945.

A personal note on further investigations

Almost all of my information on Mihály Bauer came from one article written eight years after his death by his student, László Rédei. I have not found any other biographical information about Bauer, even though he was a teacher of such well-known mathematicians as Pál Erdős and Pál Turán. Some of his mathematics was noted by Barna Szénássy *SzeBar1*, though almost no attention was paid to his contributions in number theory. It is my hope to become much more acquainted with Bauer's life and work, possibly writing a more mathematically oriented article in the future.

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